Diamagnetic limit of superconductivity with triplet pairing

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A critical magnetic field which is peculiar to superconductors with pairs that have a nonzero orbital angular momentum has been found. This critical field stems from the local orbital diamagnetism of the pairs. It agrees in order of magnitude with the paramagnetic limit for superconductors with singlet pairing.

In type II superconductors with high upper critical fields, an important depairing effect is exerted by a magnetic field on the spins of electrons in Cooper pairs. The limiting field H_p—the so-called paramagnetic limit—which destroys the superconductivity, is determined from the equality of the energy of the superconducting ordering to the energy loss of the spin polarization of the superconductor in comparison with a normal metal[1,2]: $\frac{1}{2}(\chi_n - \chi_S)H^2$, where χ_n and χ_s are the paramagnetic susceptibilities of the normal metal and of the superconductor. For a pure superconductor with singlet pairing at $T=0$ we would have $\chi_s = 0$, and $H_p = \sqrt{2}\Delta/\mu_B$, so we find $H_p/T_c = 18400$ G/K. Here Δ is the energy gap at $T=0$, μ_B is the Bohr magneton, and T_c is the critical temperature. In superconductors with paramagnetic impurities, spin-orbit scattering[3] leads to a nonzero susceptibility χ_s $(T=0)$, so that the paramagnetic limit becomes substantially higher.

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For superconductors with triplet pairing (the spin of the pair is $S=1$) or, more precisely, for those superconductors with triplet pairing in which the spin state of the Cooper pairs is an equiprobable combination of states with pair spin projections $S_z \pm 1$ on to the quantization axis [such as the A phase or the planar phase in superfluid ^{3}He (Ref. 4)], the spin susceptibility is the same as the paramagnetic susceptibility of the normal metal,[1)] so that there is no paramagnetic limit of superconductivity ($H_p = \infty$).

It should be taken into consideration, however, that in superconductors with triplet pairing the magnetic field causes, in addition to a spin polarization, Larmour precession of Cooper pairs in states with a nonzero projection of the orbital angular momentum onto the quantization axis, with the effect that the liquid acquires a diamagnetic moment $\chi_{\rm orb} H$. The magnitude of the local diamagnetic susceptibility can be estimated from

$$\chi_{\rm orb} \approx -n_s \frac{e^2}{m^* c^2} \xi^2 \left(\frac{\Delta}{\epsilon_F}\right)^2$$

where n_s is the density of the superfluid component, $e^2/m^* c^2 \xi^2$ is the diamagnetic susceptibility of one Cooper pair, m^* is the effective mass of the electron, $\xi = \hbar v_F/\Delta$ is the coherence length, and the factor $(\Delta/\epsilon_F)^2$ is the attenuation of the local susceptibility of the liquid due to the strong overlap of Cooper pairs, as in the case of the internal angular momentum in ^{3}He-A (Ref. 5). We thus find $\chi_{\rm orb} \approx -(n_s/n) N_0 \mu_B^2 (m/m^*)^2$, where N_0 is the state density at the Fermi surface; i.e., the susceptibility at $T=0$ is of the same order of magnitude as the spin paramagnetic susceptibility $\chi_n = N_0 \mu_B^2$. We thus see that in superconductors with triplet pairing there is an excess diamagnetic polarization energy $-\frac{1}{2}\chi_{\rm orb} H^2$, ("excess" in comparison with the normal metal). This circumstance is equivalent to a loss in the energy of the spin polarization in superconductors with a singlet pairing. The existence of superconductivity with triplet pairing is thus limited by a field $H_d \sim (\Delta_0/\mu_B)(m^*/m)$, which might be called the "diamagnetic limit of superconductivity."

At first glance, the presence of a factor m^*/m, which can reach a value of 10^2–10^3 in heavy-fermion superconductors,[6] would appear to mean that H_d is extremely high. However, measurements of the field penetration $\lambda = (m^* c^2/4\pi n_s e^2)^{1/2}$ in UBe_{13} yield[7] $\lambda \approx 3.6 \times 10^{-5}$ cm, i.e., a value which is typical in order of magnitude of ordinary superconductors. This circumstance can be explained on the basis of the participation of a light electron component also, with $m^* \simeq m$, in the superconductivity of this material. If this is the case, then the diamagnetic limit is on the order of the paramagnetic limit. At the same time, the upper critical fields H_{c2} in heavy-fermion supercondcutors (Ref. 8, for example) are well above the paramagnetic limit. Consequently, the existence of a diamagnetic limit renders problematical an explanation of the superconductivity in heavy-fermion compounds in terms of Cooper pairing in a triplet state.

We turn now to a formal calculation of $\chi_{\rm orb}$. For this purpose, we write the Gor'kov equations (the notation is that of Ref. 9) in the spatially uniform case, ignoring effects associated with a crystalline anisotropy and a spin-orbit coupling:

$$\left\{ i\omega + \mu - \frac{1}{2m^*}\left(\mathbf{k} - im\mu_B\left[\mathbf{H}\frac{\partial}{\partial \mathbf{k}}\right]\right)^2 - \mu_B \vec{\sigma}\mathbf{H} \right\} \bar{G}_\omega(\mathbf{k}) - \bar{\Delta}(\mathbf{k})\bar{F}^+_\omega(\mathbf{k}) = 1\,,$$

$$\left\{ - i\omega + \mu - \frac{1}{2m^*}\left(\mathbf{k} - im\mu_B\left[\mathbf{H}\frac{\partial}{\partial \mathbf{k}}\right]\right)^2 - \mu_B \vec{\sigma}\mathbf{H} \right\} \bar{F}^+_\omega(\mathbf{k}) + \bar{\Delta}^+(\mathbf{k})\bar{G}_\omega(\mathbf{k}) = 0\,. \tag{1}$$

The terms containing the operator

$$\hat{L} = - i\frac{m}{m^*}\mu_B \mathbf{H}\left[\mathbf{k}\frac{\partial}{\partial \mathbf{k}}\right]$$

are responsible for the local orbital magnetism. Finding the correction of second order in $\hat{L}$ to the Green's function in the absence of a magnetic field (the first-order correction gives a small spontaneous magnetic moment[9]), we can calculate the diamagnetic component of the free energy of the system[10]:

$$\mathscr{F}_{\rm orb} = \frac{V}{2}\,\mathrm{Sp}T\sum_{\omega,\mathbf{k}} i\int_{\omega}^{\infty} \bar{G}_2(\omega',\mathbf{k})\,d\omega' + \mathrm{H.a.} \tag{2}$$

Here V is the volume of the superconductor. Going through the necessary calculations, we find

$$\mathscr{F}_{\rm orb} = V\,\mathrm{Sp}T\sum_{\omega,\mathbf{k}} \frac{-3(\hat{L}\Delta)(\hat{L}\Delta^+) + \hat{L}^2(\Delta\Delta^+)}{12D^2} - \frac{(\hat{L}(\Delta\Delta^+))^2}{12D^3}$$

$$D = \omega^2 + \xi^2 + \Delta\Delta^+\,. \tag{3}$$

To pursue the calculations, it is convenient to specify the particular form of $\Delta(\mathbf{k}) = i(\mathbf{d}\vec{\sigma})\sigma_y$. For a superconductor with the structure of the B phase of ^{3}He, for example, with[4] $d_\alpha = \Delta_0 R_{\alpha i} k_i$, we find

$$\chi^B_{\rm orb} = -\frac{1}{V}\frac{\partial^2 \mathscr{F}_{\rm orb}}{\partial H^2} = -\frac{1}{6}\mu_B^2 N_0\left(\frac{m}{m^*}\right)^2(1 - Y(T))\,, \tag{4}$$

where $Y(T)$ is the Yosida function $[Y(0) = 0,\ Y(T_c) = 1]$. We thus see that $\chi^B_{\rm orb}$ agrees with the estimate above.

For anisotropic phases, the calculations and the ultimate result, although not complicated, are quite lengthy because of the tensor nature of the susceptibility and because $\hat{L}(\Delta\Delta^+) \neq 0$. We simply note that for the A phase with $d_\alpha = \Delta_0 V_\alpha \Delta_i \hat{k}_i$ ($\mathbf{V}^2 = 1$; $\vec{\Delta}\vec{\Delta} = 0$, $\vec{\Delta}\vec{\Delta}^* = 2$), in which the field is parallel to the orbital angular momentum $\mathbf{l} = (i/2)[\vec{\Delta}\vec{\Delta}^*]$ at $T = 0$, we find the natural result $\chi^A_{\rm orb} = \frac{3}{2}\chi^B_{\rm orb}$, since in the B phase only two-thirds of the Cooper pairs are in states with a nonzero angular-momentum projection. We also note that for a field making an angle with the $\mathbf{l}$ axis the orbital susceptibility of the A phase diverges logarithmically, $\chi^A_\perp \sim \mu_B^2 N_0 \ln \Delta/T$, in the limit $T \to 0$, because of zeros in the energy gap.[2)] Superconductors with the A-phase structure thus have an extremely small diamagnetic limit as $T \to 0$ for fields making an angle with the $\mathbf{l}$ axis.

We note in conclusion that Scharnberg and Klemm[13] have calculated H_{c2} in superconductors with triplet pairing at an arbitary temperature, but without consideration of the crystalline symmetry of the order parameter in real materials. The local orbital magnetism accordingly dropped out of the picture, since in the solution of the linear integral equation for Δ, which determines H_{c2}, only the gradients of the order parameter $\Delta(\mathbf{r},\mathbf{r}')$ along the "slow coordinate" $(\mathbf{r}+\mathbf{r}')/2$ of the center of mass of the Cooper pair were taken into account—not the dependence of Δ on $\mathbf{r}-\mathbf{r}'$, which is actually responsible for the local orbital magnetism in superconductors with a triplet pairing. The incorporation of this dependence leads to a certain renormalization of H_{c2} because of the orbital susceptibility at all temperatures. In particular, near T_c, the Ginzburg-Landau expansion[14-16] acquires terms of the type $N_0\mu_B^2(\Delta/T_c)^2$, not only because of the difference between the paramagnetic susceptibility of the superconductor and the susceptibility of the normal metal[17] but also because of the orbital susceptibility. A calculation of the upper critical field in a real superconductor with heavy fermions will of course require an understanding of the superconductivity in these materials. As yet we do not have such an understanding.

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[1)] Here we have in mind the maximum value of the susceptibility along the spin-quantization axis. We are also ignoring the tensor nature of the susceptibility, which is of course important if the spin-orbit coupling in a real crystal is taken into account.

[2)] A behavior of this sort is also characteristic of the dynamic orbital susceptibility of ^{3}He-A (Refs. 11 and 12).

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